Aerodynamic Coupling between Lateral and Longitudinal Degrees of Freedom

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An oscillatory technique for routine measurement of the direct, cross, and cross-coupling moment derivatives due to pitching and yawing has been developed and a series of comprehensive results at Mach numbers 0.7 and 0.25 and at angles of attack up to 40 deg has been obtained in a large wind tunnel. It was found that some of the dynamic cross-coupling derivatives, which at low angles of attack were all insignificant, at higher angles of attack could reach values that rendered the pertinent cross-coupling terms in the equations of motion comparable in magnitude to the well-established damping terms. This applied in particular to results obtained at Mach number 0.7. Large nonlinear variations with angle of attack were observed in many of the measured derivatives including the important damping-in-pitch and damping-in-yaw derivatives. It was concluded that it may be desirable to include certain dynamic cross-coupling derivatives in the equations of motion, and to consider all of these equations simultaneously, rather than in separate longitudinal and lateral groups.

Nomenclature

b = wingspan, 20.6 cm (8.1 in.)
$\bar{c}$ = wing mean aerodynamic chord, 7.44 cm (2.93 in.)
C_ℓ = $L/(\bar{q}Sb)$
C_m = $M/(\bar{q}S\bar{c})$
C_n = $N/(\bar{q}Sb)$
I = moment or product of inertia
K = angular stiffness
L = rolling moment
M = pitching moment; Mach number
N = yawing moment
p = angular velocity in roll
q = angular velocity in pitch
$\bar{q}$ = dynamic pressure
r = angular velocity in yaw
$Re_{\infty,\ell}$ = freestream Reynolds number based on body length
S = gross wing area, 133.3 cm^2 (20.67 in.^{2})
t = time
V = freestream velocity
x,y,z = orthogonal system of axes with origin in the aircraft center of gravity and with x axis pointing forward in the body
α = incremental angle of attack
α_0 = mean angle of attack
β = incremental angle of sideslip
γ = mechanical damping
η = phase angle
θ = angular deflection in pitch
μ = damping coefficient
ϕ = angular deflection in roll
ψ = angular deflection in yaw
ω = (circular) frequency

Subscripts

θ,ϕ,ψ = oscillation in pitch, roll or yaw
x,y,z = x, y, or z axis or direction
$\alpha,\beta,\dot\alpha,\dot\beta,p,q,r$ = derivative (of a moment) with respect to α, β, etc.

Superscripts

$\dot{}$ = time derivative

Derivatives

$$C_{\ell\alpha}=\frac{\partial C_\ell}{\partial\alpha}\qquad C_{m\alpha}=\frac{\partial C_m}{\partial\alpha}\qquad C_{n\alpha}=\frac{\partial C_n}{\partial\alpha}$$

$$C_{\ell\beta}=\frac{\partial C_\ell}{\partial\beta}\qquad C_{m\beta}=\frac{\partial C_m}{\partial\beta}\qquad C_{n\beta}=\frac{\partial C_n}{\partial\beta}$$

$$C_{\ell\dot\alpha}=\frac{\partial C_\ell}{\partial(\dot\alpha\bar{c}/2V)}\qquad C_{m\dot\alpha}=\frac{\partial C_m}{\partial(\dot\alpha\bar{c}/2V)}\qquad C_{n\dot\alpha}=\frac{\partial C_n}{\partial(\dot\alpha\bar{c}/2V)}$$

$$C_{\ell\dot\beta}=\frac{\partial C_\ell}{\partial(\dot\beta b/2V)}\qquad C_{m\dot\beta}=\frac{\partial C_m}{\partial(\dot\beta b/2V)}\qquad C_{n\dot\beta}=\frac{\partial C_n}{\partial(\dot\beta b/2V)}$$

$$C_{\ell q}=\frac{\partial C_\ell}{\partial(q\bar{c}/2V)}\qquad C_{mq}=\frac{\partial C_m}{\partial(q\bar{c}/2V)}\qquad C_{nq}=\frac{\partial C_n}{\partial(q\bar{c}/2V)}$$

$$C_{\ell r}=\frac{\partial C_\ell}{\partial(rb/2V)}\qquad C_{mr}=\frac{\partial C_m}{\partial(rb/2V)}\qquad C_{nr}=\frac{\partial C_n}{\partial(rb/2V)}$$

Note: All stability derivatives are referred to a system of body axes. For oscillation around a fixed body axis at an angle of attack α_0 the dynamic derivatives appear in the following combinations:

$$C_{\ell q}+C_{\ell\dot\alpha}\qquad C_{mq}+C_{m\dot\alpha}\qquad C_{nq}+C_{n\dot\alpha}$$

$$C_{\ell r}-C_{\ell\dot\beta}\cos\alpha_0\qquad C_{mr}-C_{m\dot\beta}\cos\alpha_0\qquad C_{nr}-C_{n\dot\beta}\cos\alpha_0$$

Introduction

THE modern military aircraft often flies at angles of attack high enough to cause significant nonlinear effects involving phenomena such as separated flows, vortex shedding, vortex bursts, etc.[1] It also flies at combinations of moderate angle of attack and a small angle of sideslip, e.g., as a result of applying direct side-force controls. The resulting irregular and often asymmetric flow phenomena that occur at these flight conditions not only cause the principal one-degree-of-freedom static and dynamic derivatives to vary dramatically with angle of attack, but also introduce certain aerodynamic coupling effects that were completely negligible

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at lower angles of attack and symmetric flow conditions of interest in the past. Static and dynamic terms representing the lateral aerodynamic moments due to longitudinal angular motions (and vice-versa) become of importance and may have to be properly taken into account in the equations of motion.

In this paper the fluid dynamic background of the problem will first be discussed, based on a flow visualization study on an aircraft-like model configuration. This will be followed by formulation of the mathematical relations on which wind-tunnel techniques for the determination of cross-coupling effects on a captive model have to be based. A recently developed dynamic cross-derivative apparatus will then be briefly described and, finally, some of the results obtained with this apparatus in the 6-ft × 6-ft wind tunnel at NASA Ames Research Center will be presented and their significance discussed.

Fluid Dynamics Considerations

To gain some understanding of the fluid dynamics phenomena that may be responsible for aerodynamic cross-coupling at higher angles of attack, the surface flow on an aircraft-like configuration was studied using an oil flow visualization technique.[2] In Fig. 1. this surface flow is shown for angle of sideslip of 10 deg and at a Mach number of 0.7. One can see one of the primary separation lines moving from one side of the model to the other as the angle of attack increases from 12.5 deg to 14 deg and then to 17.5 deg. This, of course, indicates a corresponding movement of one of the forebody vortices, which is located just above and a little to the side of the separation line. It can easily be appreciated that, if the same model were performing an oscillation in pitch around a mean angle of attack of 15 deg and with an amplitude of ±2.5 deg, the vortex would be oscillating laterally to and fro, thereby causing lateral aerodynamic reactions as functions of the angle of attack. Furthermore, due to finite flow convection velocity, the motion of the vortex in the downstream area of the model will lag the motion of the model. Therefore, lateral aerodynamic reactions that are both in phase and out of phase with the model motion can be expected to materialize, giving rise to both static and dynamic derivatives of the yawing and rolling moments and the side force, with respect to pitching. An additional, and at least as important, contribution to these derivatives is provided by the highly dissimilar flow over the two wings (and highly dissimilar changes in this flow with angle of attack) as also shown in Fig. 1. And, of course, once the flow becomes basically asymmetric, there is no longer any reason why significant derivatives should not also exist in the opposite direction, i.e., derivatives of the longitudinal forces and moments due to a lateral motion, such as static and dynamic pitching moment derivatives due to yawing. All such derivatives provide aerodynamic cross-coupling between the lateral and longitudinal degrees of freedom of an aircraft and we will call them therefore *cross-coupling* derivatives (reserving the name of *cross* derivatives for the traditional derivatives relating the two *lateral* degrees of freedom, such as the rolling moment derivative due to yawing or vice versa).

Equations of Motion

Let us now consider the equations of motion of an aircraft. An examination of these equations may prove useful to gain a better appreciation of the effects of the cross-coupling derivatives on the flight behavior of an aircraft. It will also provide the necessary basis for the development of expressions for evaluation of the special oscillatory experiments that are needed to determine these derivatives on a captive model in the wind tunnel. For simplicity, let us consider a symmetrical aircraft (with its plane of symmetry coinciding with the XZ plane and the Y axis perpendicular to it), with controls locked, and let us neglect all of the second-order terms. The equations of motion for the angular degrees of freedom of interest here and relative to a set of moving (Eulerian) axes, that are fixed in the aircraft, can be written as follows (see, for example, Ref. 3):

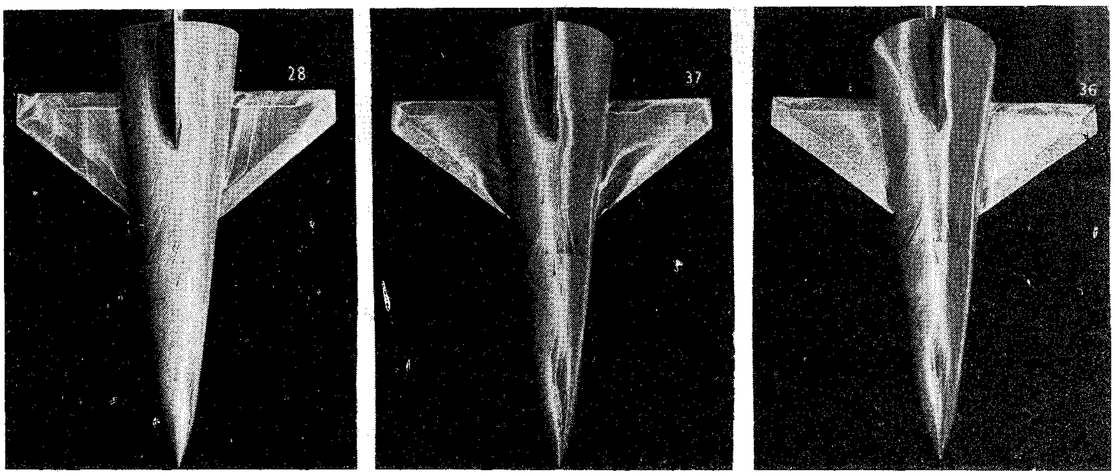
$$\Sigma L = \dot{p} I_x - \dot{r} I_{xz} \tag{1a}$$

$$\Sigma M = \dot{q} I_y \tag{1b}$$

$$\Sigma N = \dot{r} I_z - \dot{p} I_{xz} \tag{1c}$$

where the left side of each equation represents the summation of all the changes in external moments acting on the aircraft in that particular degree of freedom, due to a very small departure from the steady-state motion. For a captive aircraft model oscillating in a wind tunnel, each of these external moments consists of three distinct parts, namely, 1) change in aerodynamic reaction resulting from model oscillation, 2) change in mechanical reaction caused by model oscillation, and 3) forcing moment (if any).

For a model at an equilibrium angle of attack α_0 performing a small-amplitude low-frequency angular oscillation, the aerodynamic reactions can be expressed by a linear superposition of contributions caused by two angular displacement variables α and β, two time rates-of-change $\dot{\alpha}$ and $\dot{\beta}$ of these displacement variables and three angular



$\alpha = 12.5$ deg, $\beta = 10$ deg $\alpha = 14$ deg, $\beta = 10$ deg $\alpha = 17.5$ deg, $\beta = 10$ deg

Fig. 1 Surface flow on a schematic aircraft configuration at $M = 0.7$ and $Re_{\infty,\ell} = 3.3 \times 10^6$.

velocities p, q, and r. There is no need to include separately contributions caused by the displacement in roll, since at zero incidence the aerodynamic reactions are independent of the roll angle ϕ and since, at nonzero incidence and for small roll angles, the effect of roll displacement can be expressed as a modification of the true value of the angle-of-sideslip β, while the angle of attack remains approximately unchanged. Using Ref. 4 and employing a system of body axes we can write:

$$\beta = \sin^{-1}(\sin\alpha_0 \sin\phi \cos\psi - \cos\alpha_0 \sin\psi) \tag{2}$$

For small values of ϕ and ψ this relation becomes

$$\beta = \phi \sin\alpha_0 - \psi \cos\alpha_0 \tag{3}$$

The change in aerodynamic reactions due to model oscillations in ϕ, θ, and ψ can now be formulated as follows (taking the pitching moment as an example):

$$M_{\text{aero}} = M_\alpha \alpha + M_\beta \beta + M_{\dot\alpha} \dot\alpha + M_{\dot\beta} \dot\beta + M_p p + M_q q + M_r r \tag{4}$$

For a wind-tunnel model performing angular oscillations around a set of three fixed axes we have $\alpha = \theta$, $\dot\phi = p$, $\dot\alpha = q$, and $\dot\psi = r$. Inserting Eq. (3) and its time derivative, we can now rewrite Eq. (4):

$$M_{\text{aero}} = M_\beta \phi \sin\alpha_0 + M_\alpha \theta - M_\beta \psi \cos\alpha_0 + (M_p + M_{\dot\beta} \sin\alpha_0) p$$
$$+ (M_q + M_{\dot\alpha}) q + (M_r - M_{\dot\beta} \cos\alpha_0) r \tag{5}$$

A captive model performing an oscillatory motion experiences mechanical reactions from its support. If the model is mounted in such a way that no friction occurs during its motion, these mechanical reactions are limited to those caused by the mechanical stiffness K and those caused by the mechanical damping γ of the support. Denoting the angular degree of freedom in pitch by subscript θ we can describe the mechanical reactions due to oscillation in pitch of the model as $M_{\text{mech}} = -(\gamma_\theta q + K_\theta \theta)$, where the negative sign reflects the fact that standard definitions have been used for γ and K and that the damping and stiffness terms oppose the motion and the displacement, respectively.

If forcing moment (torque) in pitch is present, it can be denoted by M_T. Adding all of the external moments, inserting them together with Eq. (5) in Eq. (1b), and performing similar operations for the angular degrees of freedom in roll and in yaw, we obtain the following equations of angular motion for a captive symmetrical aircraft model:

$$I_x \dot p + [\gamma_\phi - (L_p + L_{\dot\beta} \sin\alpha_0)] p + (K_\phi - L_\beta \sin\alpha_0) \phi$$
$$= I_{xz} \dot r + (L_r - L_{\dot\beta} \cos\alpha_0) r - L_\beta \psi \cos\alpha_0$$
$$+ (L_q + L_{\dot\alpha}) q + L_\alpha \theta + L_T \tag{6a}$$

$$I_y \dot q + [\gamma_\theta - (M_q + M_{\dot\alpha})] q + (K_\theta - M_\alpha) \theta$$
$$= (M_r - M_{\dot\beta} \cos\alpha_0) r - M_\beta \psi \cos\alpha_0 + (M_p + M_{\dot\beta} \sin\alpha_0) p$$
$$+ M_\beta \phi \sin\alpha_0 + M_T \tag{6b}$$

$$I_z \dot r + [\gamma_\psi - (N_r - N_{\dot\beta} \cos\alpha_0)] r + (K_\psi + N_\beta \cos\alpha_0) \psi$$
$$= I_{xz} \dot p + (N_p + N_{\dot\beta} \sin\alpha_0) p + N_\beta \phi \sin\alpha_0$$
$$+ (N_q + N_{\dot\alpha}) q + N_\alpha \theta + N_T \tag{6c}$$

It should be noted that all the stability derivatives appearing in the preceding equations are *local* derivatives, that is, derivatives applicable to the particular equilibrium position (α_0, β_0) of the model, and that this formulation is valid, in a strict sense, only for small values of β_0 and for very small amplitude oscillations that do not extend over any discontinuities in the aerodynamic characteristics. The aerodynamic terms on the right-hand side of Eqs. (6) represent coupling effects that are similar to the "interaction terms" in the representation of the aerodynamic moments proposed in Ref. 5.

Equations (6) can be used to obtain the *direct* static and dynamic derivatives, when their left-hand side is equated to a driving torque (such as N_T) in the same degree of freedom, or to obtain the *cross* and *cross-coupling* static and dynamic derivatives, by examining their right-hand side in relation to a driving motion in another degree of freedom. The former follows a straightforward classical procedure and need not be dwelled on here. The latter will be discussed by considering, for example, the induced secondary oscillation in pitch caused by a primary sinusoidal oscillation in yaw, $\psi = |\psi| e^{i\omega t}$. As indicated in Eq. (6b), a primary motion in yaw may cause certain cross-coupling effects on the secondary oscillation in pitch. These effects are represented by terms in r and ψ on the right-hand side of Eq. (6b). The pitching moment $\bar M$ induced by this cross-coupling is synchronous with the primary oscillation in yaw but, in general, is slightly shifted in phase by a quantity η_θ. We can write

$$\bar M = |\bar M| e^{i(\omega t + \eta_\theta)} = (M_r - M_{\dot\beta} \cos\alpha_0) r - M_\beta \psi \cos\alpha_0 \tag{7}$$

Dividing Eq. (7) successively by ψ and $\dot\psi$ we obtain

$$\bar M = \psi \left| \frac{\bar M}{\psi} \right| e^{i\eta_\theta} \tag{8}$$

and

$$\bar M = \dot\psi \frac{\bar M}{i\omega |\psi|} e^{i\eta_\theta} \tag{9}$$

Differentiating Eqs. (7-9) with respect to ψ and $\dot\psi$ and using the real parts of the resulting expressions we get

$$\frac{\partial \bar M}{\partial \psi} = \left| \frac{\bar M}{\psi} \right| e^{i\eta_\theta} \rightarrow \frac{|\bar M| \cos\eta_\theta}{|\psi|} = -M_\beta \cos\alpha_0 \tag{10}$$

$$\frac{\partial \bar M}{\partial \dot\psi} = \frac{\bar M}{i\omega |\psi|} e^{i\eta_\theta} \rightarrow \frac{|\bar M| \sin\eta_\theta}{\omega |\psi|} = \frac{\partial \bar M}{\partial r} = M_r - M_{\dot\beta} \cos\alpha_0 \tag{11}$$

Equations (10) and (11) lead to the same expressions for dimensionless cross-coupling derivatives as already derived in Ref. 6, except that $C_{m\beta}$ on line 9, left-hand column, p. 8, should be multiplied by $\cos\alpha_0$; this factor was inadvertently omitted in Ref. 6. The experimental method for determining $\bar M$ and η_θ is described in Ref. 7.

The remaining cross and cross-coupling derivatives can be obtained in a similar fashion. Table 1 gives all of the expressions for the static and dynamic moment derivatives in one degree of freedom (DOF) due to an angular oscillation in a second DOF, where η denotes the phase angle between the induced moment in the secondary DOF and the displacement in the primary DOF, the former being indicated by the subscript.

When examining the left-hand side of Eqs. (6) it can also be seen that the nonzero equilibrium incidence will affect the determination of the direct moment derivatives in yaw and in roll. For a single-degree-of-freedom angular oscillation the static and dynamic derivatives can be obtained by applying standard procedures analogous to those required for oscillation-in-pitch data evaluation, as indicated in Table 2.

Oscillatory Experiments

The most direct method to determine cross-coupling derivatives experimentally is to impose a known oscillatory motion in one degree of freedom and to measure the resulting

Table 1 Cross and cross-coupling derivatives

DOF primary/ secondary	Static derivative	Dynamic derivative
Yaw/pitch	$-M_\beta \cos\alpha_0 = \dfrac{\lvert\bar{M}\rvert \cos\eta_\theta}{\lvert\psi\rvert}$	$M_r - M_{\dot\beta}\cos\alpha_0 = \dfrac{\lvert\bar{M}\rvert \sin\eta_\theta}{\omega \lvert\psi\rvert}$
Yaw/roll	$-L_\beta \cos\alpha_0 = \dfrac{\lvert\bar{L}\rvert \cos\eta_\phi}{\lvert\psi\rvert}$	$L_r - L_{\dot\beta}\cos\alpha_0 = \dfrac{\lvert\bar{L}\rvert \sin\eta_\phi}{\omega \lvert\psi\rvert}$
Pitch/yaw	$N_\alpha = \dfrac{\lvert\bar{N}\rvert \cos\eta_\psi}{\lvert\theta\rvert}$	$N_q + N_{\dot\alpha} = \dfrac{\lvert\bar{N}\rvert \sin\eta_\psi}{\omega \lvert\theta\rvert}$
Pitch/roll	$L_\alpha = \dfrac{\lvert\bar{L}\rvert \cos\eta_\phi}{\lvert\theta\rvert}$	$L_q + L_{\dot\alpha} = \dfrac{\lvert\bar{L}\rvert \sin\eta_\phi}{\omega \lvert\theta\rvert}$
Roll/pitch	$M_\beta \sin\alpha_0 = \dfrac{\lvert\bar{M}\rvert \cos\eta_\theta}{\lvert\phi\rvert}$	$M_p + M_{\dot\beta}\sin\alpha_0 = \dfrac{\lvert\bar{M}\rvert \sin\eta_\theta}{\omega \lvert\phi\rvert}$
Roll/yaw	$N_\beta \sin\alpha_0 = \dfrac{\lvert\bar{N}\rvert \cos\eta_\psi}{\lvert\phi\rvert}$	$N_p + N_{\dot\beta}\sin\alpha_0 = \dfrac{\lvert\bar{N}\rvert \sin\eta_\psi}{\omega \lvert\phi\rvert}$

Table 2 Direct derivatives

DOF	Static derivative	Dynamic derivative
Pitch	M_α	$M_q + M_{\dot\alpha}$
Yaw	$-N_\beta \cos\alpha_0$	$N_r - N_{\dot\beta}\cos\alpha_0$
Roll	$L_\beta \sin\alpha_0$	$L_p + L_{\dot\beta}\sin\alpha_0$

aerodynamic reactions in one or more of the other degrees of freedom. Separate apparatuses are therefore required to measure cross and cross-coupling derivatives due to oscillatory pitching (or yawing), oscillatory rolling, and oscillatory translation (in the vertical or lateral direction). However, it is also relatively easy to incorporate in each apparatus the ability to measure the direct derivatives in its primary degree of freedom. Since cross-coupling derivatives are expected to be significant only at moderate to high angles of attack and possibly also at some finite angles of sideslip, it is important to design all such apparatuses to be compatible with the high aerodynamic loads normally associated with those flow conditions.

In this paper a series of experiments is described that was conducted to determine all the cross and cross-coupling moment derivatives due to oscillatory pitching and yawing. At the same time, the corresponding direct moment derivatives were also obtained. A cross-derivative apparatus developed for these experiments is shown in the schematic drawing in Fig. 2. The apparatus provides a primary oscillatory motion in pitch with resulting secondary oscillatory motions in yaw and roll or, alternatively—with the balance rotated 90 deg around its longitudinal axis—a primary oscillatory motion in yaw with resulting secondary motions in pitch and roll. In each case the components of the two secondary motions that are in-phase and out-of-phase with the primary motion are measured and converted, by a rather complex procedure, into the in-phase and out-of-phase induced aerodynamic moments in the secondary degrees of freedom. In addition, the torque, the amplitude, and the frequency of the primary motion are also measured. All together, a set of two oscillatory experiments, one in pitch and one in yaw, is required to obtain a complete set of four static and four dynamic cross and cross-coupling moment derivatives as well as two static and two dynamic direct moment derivatives. This includes the traditional cross derivative of the rolling moment due to yawing and the well-known damping-in-pitch and damping-in-yaw derivatives. All of the derivatives are measured in a system of body axes. A more detailed discussion of the apparatus, the associated instrumentation system, and the data reduction procedure is given in Ref. 7. This reference also includes a description of a special three-degrees-of-freedom dynamic calibrator which was developed and used—with very good results—to verify the experimental method and especially the data reduction procedure.

A series of dynamic stability experiments was carried out in the NASA Ames Research Center's 6-ft×6-ft wind tunnel using the experimental technique and the cross-derivative apparatus described (Fig. 3). The experiments were performed with the same aircraft-like model (Fig. 4) as the one previously used for the surface-flow visualization experiments. The measurements were made at Mach numbers of 0.25 and 0.7 and at sideslip angles −5, 0, and 5 deg. The angle of attack was remotely adjustable in the range of 0 to 40 deg in 1-deg intervals and all of the results were automatically processed and plotted after each pair of pitching and yawing runs. The Reynolds number was 1.5×10^6/ft and 3.6×10^6/ft at Mach numbers 0.25 and 0.7, respectively. Frequencies of the primary motion were 38 Hz in pitch and 36 Hz in yaw, resulting in reduced frequencies ($\omega\ell/2V$) ranging from 0.04 to 0.11. The natural frequencies in the secondary degrees of freedom were approximately 57 Hz in yaw, 63 Hz in pitch, and 84 Hz in roll. The amplitude of the primary motion was 1 deg. The axis of oscillation and the point around which pitching and yawing moments were measured was at a

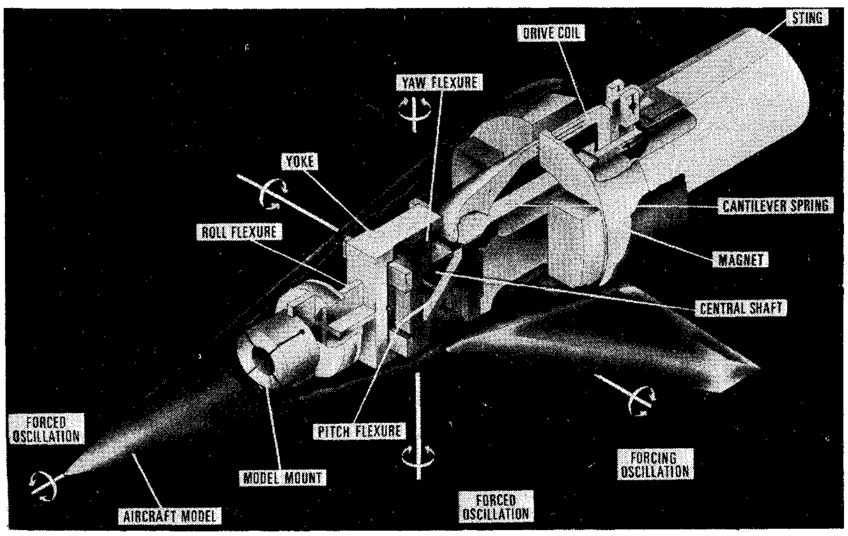


Fig. 2 Cross-derivative apparatus in the pitching mode.

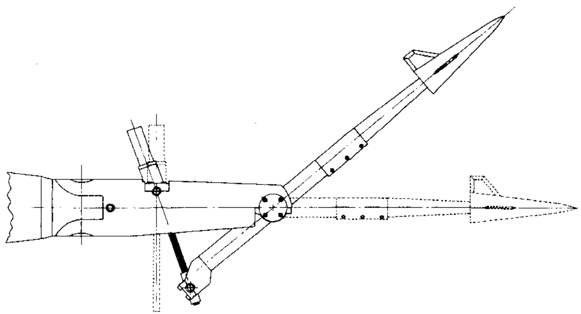


Fig. 3 Model installation in NASA Ames' 6-ft × 6-ft wind tunnel.

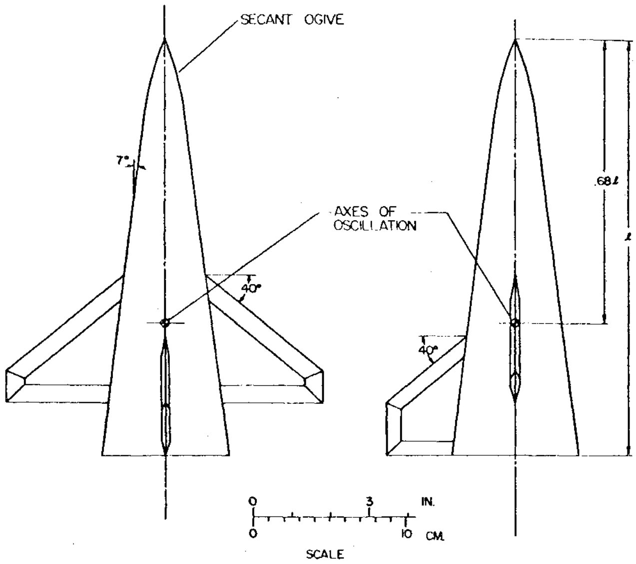


Fig. 4 Model geometry.

distance of 0.68 of the body length from the nose of the model.

It should be noted that the model installation shown in Fig. 3 includes a stiff front sting and a relatively massive aft sting. As determined by accelerometers mounted on the front sting inside the model, sting oscillation in the plunging degree of freedom was negligible. The lowest natural frequency in pitch of the sting (with the model on) was 54 Hz; that is well above the frequency of the primary motion in pitch. Due to the larger sting flexibility in yaw (caused by the angle-of-attack change mechanism), the sting frequency in yaw was lower than in pitch, introducing the possibility of small lateral sting oscillation. This effect was not corrected for. As pointed out in Ref. 8, it is possible that at angles of attack high enough to generate forebody vortices but not high enough to cause the vortex breakaway from the body surface, a vortex burst could be caused by the aerodynamic interference with the massive aft sting and that this vortex burst could be felt as far upstream as on the model. Indeed, a comparison of the present damping-in-pitch results with similar results obtained in the NAE 30-in. wind tunnel partly with a more slender sting arrangement[6] and partly with the half-model technique (not yet published) shows excellent agreement among the three sets of results up to $\alpha_0 = 18$ deg. At higher angles of attack the slender-sting and the half-model results continue to be in excellent agreement, while the present results, although showing the same general trends, indicate a higher damping. This could probably be due to the interference with the aft sting at these intermediate angles of attack. (If so, the same interference may also affect some of the other derivatives at these values of α_0; however, the quantitative effects of that, although not expected to be large, are not easy to assess.) At $\alpha_0 = 30$ deg the three sets of results are again in close agreement.

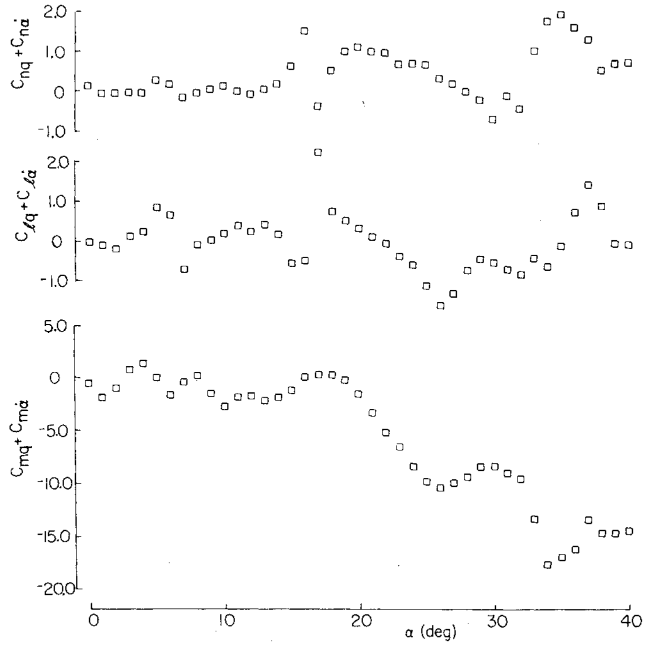


Fig. 5 Dynamic derivatives due to pitching; Mach 0.7; $\beta_0 = 5$ deg.

Experimental Results

Only a fraction of all the experimental material collected during this investigation[9] can be presented in this paper. The sequence of results obtained at Mach number 0.7 and angle of sideslip 5 deg has been selected for this purpose. In Fig. 5 the three dynamic derivatives due to pitching are plotted as functions of angle of attack α_0. With the exception of a slight irregularity at α_0 around 6 deg, all three are relatively small at α_0 up to 14 deg or so, and display various peaks and irregular behavior at higher α_0. The highest absolute values reached are of the order of 2 for the two cross-coupling derivatives and 18 for the damping-in-pitch derivative. Well-defined peaks occur at α_0 around 16 deg and again at α_0 between 34 and 37 deg. The first of these peaks occurs at a value of α_0 where, according to the previously shown surface-flow visualization results, one of the body vortices passes over the fin of the model on its way from one side of the model to the other. This peak may also be due to a change in the separated flow pattern over the windward wing (Fig. 1). In both cases, if the change is sufficiently sudden, the derivative represents a measure of an almost discontinuous change in the pertinent aerodynamic characteristic (such as the well-known "snap-roll" phenomenon) when "catching" it with an oscillation of $\Delta\alpha = 1$ deg. In such a situation the derivative can be expected to be inversely proportional to $\Delta\alpha$, i.e., it can become very large with decreasing $\Delta\alpha$, *if* the discontinuity is still contained within the range of oscillation. Between $\alpha_0 = 19$ deg and $\alpha_0 = 26$ deg all three derivatives display a smooth variation with an almost constant negative slope. The irregularities in $(C_{mq} + C_{m\dot{\alpha}})$ for $\alpha_0 < 8$ deg should probably be disregarded.

In Fig. 6 the three dynamic derivatives due to yawing are shown. The first one, a cross-coupling derivative, displays trends and variations of a similar character to those observed in the cross-coupling derivatives in Fig. 5, with a maximum absolute value of the order of 1. The second one, a "traditional" cross derivative, is relatively constant at low α_0 and then declines to practically zero at $\alpha_0 = 34$ deg. Note the magnitude of this derivative, which nowhere is greater than 0.2, i.e., one-tenth of the maximum value of the cross-coupling derivative in Fig. 5. Finally, the damping-in-yaw derivative has a constant negative value at low α_0, followed by a well-defined peak at $\alpha_0 = 16$ deg and a rather irregular behavior at higher α_0. (The same well-defined peak was also observed at $\beta_0 = 0$ deg and $\beta_0 = -5$ deg.) Both at the peak and at high α_0 the derivative reaches a value of -1.2, i.e. four times the value at low α_0.

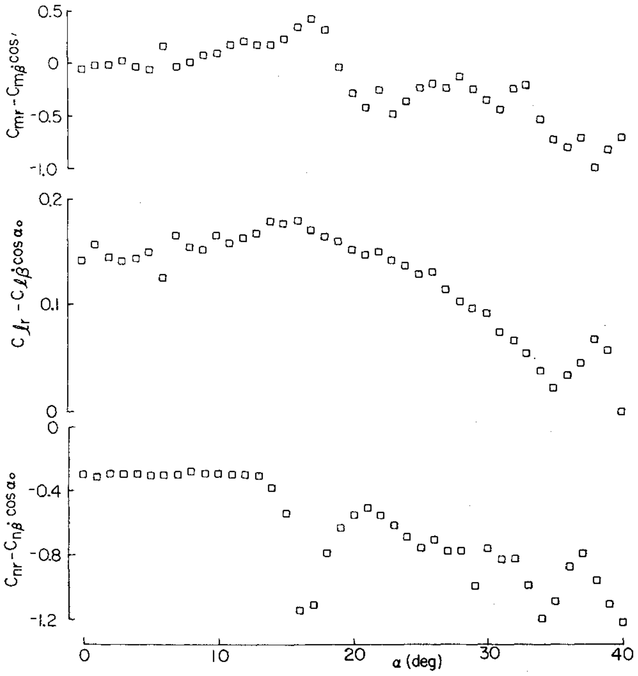


Fig. 6 Dynamic derivatives due to yawing: Mach 0.7; $\beta_0 = 5$ deg.

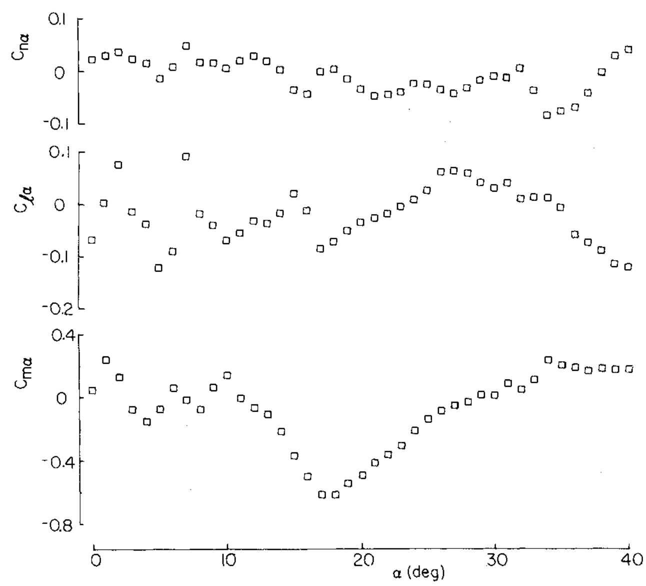


Fig. 7 Static derivatives due to pitching: Mach 0.7; $\beta_0 = 5$ deg.

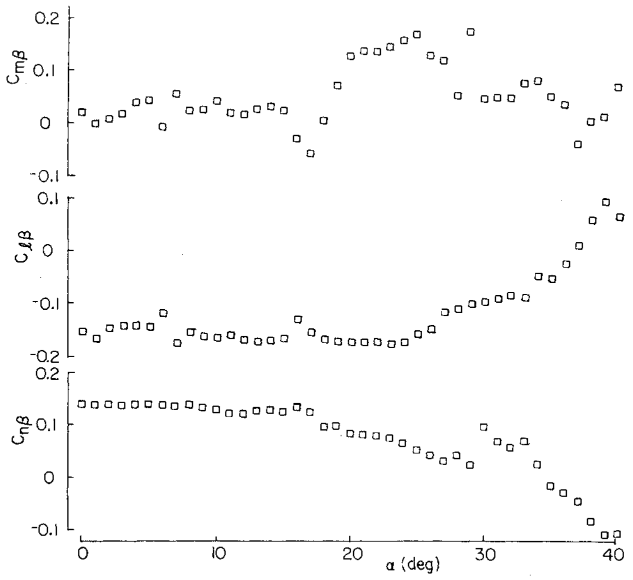


Fig. 8 Static derivatives due to yawing: Mach 0.7; $\beta_0 = 5$ deg.

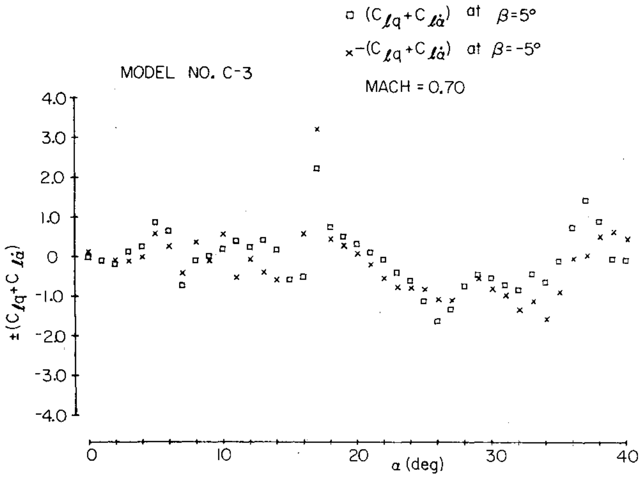


Fig. 9 Example of symmetrical results at positive and negative sideslip.

The six static derivatives due to pitching and yawing are presented in Figs. 7 and 8. The three cross-coupling derivatives are very small, their highest absolute values being of the order of 0.1 to 0.2. If multiplied by a suitable negative constant their variation with α_0 seems to follow, at higher α_0, the same pattern as that of their dynamic counterparts, which is typical for aerodynamic effects induced by flow separation and associated time-lag effects.[10] The direct pitching moment derivative displays a well-defined negative peak centered around $\alpha_0 = 17$ deg. Its small magnitude is, of course, a direct result of locating the moment reference point (and the axis of oscillation) near the aerodynamic center of the model, which was convenient from the point of view of the experiments as well as realistic for simulating the full-scale conditions. Again, the irregularities in $C_{\ell\alpha}$ and $C_{m\alpha}$ at $\alpha_0 < 8$ deg should probably be disregarded. The static cross derivative $C_{\ell\beta}$ is small and negative at α_0 up to 24 deg, tending toward slightly positive values at α_0 around 40 deg. It is of the same magnitude as the corresponding dynamic cross derivative.

The fact that the experiments were conducted at a sideslip angle of both $+5$ deg and -5 deg provided a unique opportunity for checking the consistency of the results. As could be expected from basic aerodynamic considerations, all derivatives that were "all longitudinal" or "all-lateral" were found to be nearly the same for $\beta_0 = +5$ deg and $\beta_0 = -5$ deg, while all the cross-coupling derivatives displayed nearly the same variation with α_0, but with a reversed sign. An example of the latter is shown in Fig. 9, where derivative $+(C_{\ell q} + C_{\ell\dot{\alpha}})$ at $\beta_0 = +5$ deg is compared with $-(C_{\ell q} + C_{\ell\dot{\alpha}})$ at $\beta_0 = -5$ deg. It can be seen that the various peaks and slopes are quite faithfully reproduced, with only a minor shift, probably due to β_0 not having exactly the same absolute value in the two cases.

As previously mentioned, similar experiments were also conducted at Mach number 0.25. With the exception of derivatives $C_{n\beta}$ and $C_{\ell\beta}$, all of the derivatives showed variations with α_0 that were similar in character but smaller in magnitude than those presented here. All the cross-coupling derivatives at zero sideslip were practically zero. Derivatives $C_{n\beta}$ and, to a lesser degree, $C_{\ell\beta}$ displayed a well-defined rounded peak, centered around $\alpha_0 = 20$ deg[9]; this peak is totally absent at Mach number 0.70. This difference can possibly be attributed to the fact that at Mach 0.25, due to a much lower Reynolds number, the boundary-layer transition probably occurs on the body near the model base, while at Mach 0.7, at higher Reynolds number, it occurs well forward of the fin. It has been shown in Ref. 11 that transition can interact with the fin-induced separation causing significant aerodynamic effects.

It should be noted that the present results are considerably different from the preliminary results reported in Ref. 6, which were obtained with a weaker and more flexible system, conducive to sting vibrations. In addition, those preliminary results were based on a different set of reference quantities and it was discovered that in the data analysis a factor of $360/(2\pi)$ was inadvertently omitted, rendering all the dynamic cross-coupling derivatives much too small. The preliminary results of Ref. 6 should therefore be disregarded.

Significance of Cross-Coupling Terms

Since this is the first time that dynamic cross-coupling derivatives have been systematically determined, it may be of some interest to speculate briefly on their significance when solving the equations of motion of an aircraft. Short of conducting a complete sensitivity analysis using a full set of equations of motion, some appreciation of the significance of a given cross-coupling derivative can be obtained by comparing the magnitude of the term that includes such a derivative with the magnitude of another term (in the same equation) that contains one of the traditional derivatives whose significance is well established. For example, from the earlier developed rolling-moment equation for a captive aircraft model [Eq. (6a)] we can extract the two terms representing the rolling moment due to the rate of yaw and the rate of pitch. Rewriting these terms to include the dynamic derivatives in the dimensionless form we get

$$(L_r - L_{\dot\beta}\cos\alpha_0)r + (L_q + L_{\dot\alpha})q$$
$$= (\bar{q}Sb/2V)[(C_{\ell r} - C_{\ell\dot\beta}\cos\alpha_0)br + (C_{\ell q} + C_{\ell\dot\alpha})\bar{c}q] \qquad (12)$$

Therefore the ratio of the new dynamic cross-coupling term to the "traditional" term is

$$(\bar{c}q/br)(C_{\ell q} + C_{\ell\dot\alpha})/(C_{\ell r} - C_{\ell\dot\beta}\cos\alpha_0) \qquad (13)$$

Similarly, for the other two pairs of terms we obtain, from Eqs. (6b) and (6c)

$$(br/\bar{c}q)(C_{mr} - C_{m\dot\beta}\cos\alpha_0)/(C_{mq} + C_{m\dot\alpha}) \qquad (14)$$

and

$$(\bar{c}q/br)(C_{nq} + C_{n\dot\alpha})/(C_{nr} - C_{n\dot\beta}\cos\alpha_0) \qquad (15)$$

For the present configuration the ratio $b/\bar{c} = 2.77$. Assuming that the rates of yaw and pitch are the same ($q = r$) and using values of the derivatives as reported in this paper, we find that a) the maximum value of expression (13) is 4; b) the maximum value of expression (14) is 1/7 if referred to the maximum value of the damping-in-pitch term, and 2 if referred to its value at low α_0; c) the maximum value of expression (15) is 3/5 if referred to the maximum value of the damping-in-yaw term and 3 if referred to its value at low α_0.

Since the importance of the two damping derivatives at low angles of attack has long been firmly established and the significance of the cross derivative $(C_{\ell r} - C_{\ell\dot\beta}\cos\alpha_0)$ is also becoming more and more recognized, it follows that the three cross-coupling derivatives—in cases where they approach their maximum values (i.e., at higher angles of attack)—may be of a comparable significance. This conclusion applies when $r/q = 1$, as assumed in the foregoing. If $r/q \neq 1$, it follows from expressions (13-15) that the significance of some cross-coupling derivatives will be decreased while that of some others will actually be increased. Thus, no matter what r/q is, there should always be at least one cross-coupling derivative of fairly high significance.

This line of reasoning is based on Eqs. (6) and therefore applies, in a strict sense, only to the case of a captive aircraft (model) performing angular oscillations around a fixed point. It is unfortunate that the present state of knowledge prevents us from separating out $\dot\alpha$– and $\dot\beta$-effects and applying a similar reasoning to an unrestricted aircraft. There is, however, no reason to expect that such generalization would change the preceding orders of magnitude. The reader is once more cautioned that the foregoing reasoning is somewhat speculative and has yet to be confirmed by a rigid sensitivity analysis using a full set of equations of motion.

Summary and Conclusions

An oscillatory technique for routine measurement of the direct, cross, and cross-coupling moment derivatives due to pitching and yawing has been developed at the National Aeronautical Establishment and used for a systematic investigation of these derivatives in a large wind tunnel at NASA Ames Research Center. A comprehensive set of results was obtained for an aircraft-like wing-body-fin configuration at Mach numbers 0.7 and 0.25, at sideslip angles −5, 0, and 5 deg and in the angle-of-attack range from 0 to 40 deg. The most important conclusions are:

1) Static and dynamic cross-coupling moment derivatives due to pitching and yawing have been systematically determined. It was found that these derivatives, which at low angles of attack are all insignificant, at higher angles of attack could reach values comparable to or even larger than some of the more traditional dynamic derivatives; this applied in particular at Mach number 0.7. The technique was verified by tests with an electromagnetic calibrator, and the validity of the cross-coupling results was confirmed by observing their symmetric behavior at positive and negative angles of sideslip. It was shown that, in the equations of motion for a captive model, at least some of the cross-coupling terms could be of the same magnitude as the direct damping terms of well-established significance.

2) Large nonlinear effects were observed at higher angles of attack in many static and dynamic moment derivatives due to pitching and yawing oscillation; this includes the important damping-in-pitch and damping-in-yaw derivatives, where values up to 10 times the low angle-of-attack values have been measured, and where several peaks and abrupt changes have been observed.

3) These findings seem to indicate that, for best results, at least some of the dynamic cross-coupling derivatives may have to be included in the equations of motion and that, in such a case, these equations should no longer be separated into longitudinal and lateral groups, but should all be considered simultaneously. It may also be desirable to take into account the significant variations of many derivatives with the angle of attack.

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